AD-A038 113 NORTHWESTERN UNIV EVANSTON ILL STRUCTURAL MECHANICS LAB F/G 20/11
A RAY THEORY FOR ELASTODYNAMIC STRESS INTENSITY FACTORS.(U)
MAR 77 J D ACHENBACH, A K GAUTESEN N00014-76-C-0063
UNCLASSIFIED NU-SML-TR-77-1 NL

| OF |
AD
A038113

END
DATE
FILMED
4-77

12

6

A RAY THEORY FOR ELASTODYNAMIC STRESS INTENSITY FACTORS.

9 Interim rept.,

by

10

J. D. Achenbach and A. K. Gautesen

The Technological Institute
Northwestern University, structural mech. lab.
Evanston, Illinois 60201

Office of Naval Research
15 N00014-76-C-0063

12 31p.

11 March 1977

14 NU-SML- TR No. 77-1

Approved for public release; distribution unlimited

ABSTRACT

Elastodynamic stress intensity factors generated by the interaction of wave motions with a crack are analyzed. It is shown that in an asymptotic approximation, which is valid for high frequencies, the stress intensity factors at the edge of a crack are related to the fields of incident rays by a matrix of stress intensity factor coefficients, which can be computed from canonical solutions. The canonical solutions are provided by the fields describing diffraction by a semi-infinite crack of plane body waves and plane surface waves, which are incident under an arbitrary angle with the edge of the crack. Several applications of the theory are presented. For cracks of finite length, the contributions due to the travelling back and forth of rays between the two crack tips is taken into account in a simple manner, to yield results which are in excellent agreement with numerical results obtained by other authors.

1. Introduction

A crack in a solid body gives rise to singular stresses at the edge of the crack. In a local coordinate system, with the origin on the edge of the crack, a representative stress component near the edge, say τ_θ, may be expressed in the form

$$\tau_\theta(r,\theta) = (2\pi r)^{-\frac{1}{2}} K_I T_\theta(\theta) \tag{1}$$

Here r and θ are polar coordinates in the plane normal to the edge. The geometrical features of the crack geometry, as well as the parameters describing material properties and loads, enter in the stress intensity factor K_I. For dynamic problems K_I also depends on the time t and the circular frequency ω.

There is much interest in stress intensity factors, because they are relevant to fracture mechanics considerations. Here we are interested in elastodynamic stress intensity factors generated by the interaction of wave motions with a crack. We present some general results which are valid for high frequencies under time-harmonic excitations, or for small times after the arrival of wave fronts under impact loads.

Ray theory provides a very useful method to analyze both the propagation of high frequency waves and the propagation of surfaces of discontinuity, see e.g. Refs.[1] - [3]. The general ideas of ray theory were extended by Keller [4] to analyze the far field in diffraction problems. In the present paper a ray theory for stress intensity factors is established. The theory states that in an asymptotic approximation.the stress intensity factors at a point on the edge of a crack are related to the field of the incident wave at this point by a matrix of stress intensity factor coefficients, which depends only on the direction of incidence, the frequency, and the local physical properties of the solid at the point of incidence. The incident wave may be a body wave as well as a surface wave. The matrix is independent of the curvature of both the wavefront of the incident wave and the edge of the crack, provided that these curvatures are of smaller orders in magnitude than the wavenumbers.

In Section 2, an edge region analysis is presented for incident body waves. Details are worked out for an incident ray of longitudinal motion. By matching a boundary layer solution of the diffracted field in the vicinity of the edge to the outer solution (the geometrical optics field), it is shown that the stress intensity factors generated by the incident field are related in a simple manner to the stress intensity factors for a plane longitudinal wave incident upon a semi-infinite stress-free crack. The modifications required to treat incident rays of transverse motion are obvious, and they are not discussed in detail.

For incident body waves the construction of the matrix of stress intensity factor coefficients is discussed in Section 3. The coefficients are listed in Appendix A. In Section 4 the theory is extended to elastodynamic stress intensity factors generated by surface waves which are incident on the edge via the faces of the crack.

In the last section several applications of the theory are presented, and the results are compared with those obtained by other authors.

2. Edge Region Analysis

In this section a formal proof of the theory for incident body waves is presented by a method motivated by the work of Buchal and Keller [5]. An inner solution is constructed which is asymptotically valid in a region near the edge. The leading term of this solution is then matched to the leading term of the outer solution, which is the geometrical optics part of the diffracted field.

Essential to the development of this section is the "geometrical optics" part of the diffracted field, for the case that a plane longitudinal wave is diffracted by a semi-infinite crack, whose surfaces are free of tractions. The geometry is shown in Fig.1. Omitting, here and in the sequel, the factor $\exp(-i\omega t)$, where ω is the circular frequency, the incident wave is represented by

$$\underset{\sim}{w}_{inc} = \underset{\sim}{p} \exp(ik_L \underset{\sim}{p}.\underset{\sim}{x}) \tag{2}$$

In Eq.(2), we have $k_L = \omega/c_L$, where c_L is the velocity of longitudinal waves: $c_L = [(\lambda+2\mu)/\rho]^{\frac{1}{2}}$, and $\underset{\sim}{p}$ is a unit vector which defines both the direction of propagation and of displacement. In terms of the angles shown in Fig.1 we have

$$(p_1,p_2,p_3) = (\sin\phi_L \ \cos\theta_L, \ \sin\phi_L \ \sin\phi_L, \ \cos\phi_L) \tag{3}$$

The geometrical optics part of the diffracted field has been worked out in a paper by Achenbach and Gautesen [6]. The results may be written in the form

$$\underset{\sim}{w}_{go} = \exp(ik_L p_3 x_3) \ \underset{\sim}{W}_{go}(x_1,x_2) \tag{4}$$

where

$$\underset{\sim}{W}_{go}(x_1,x_2) = \underset{\sim}{W}_{inc} \ H(\theta-\theta_L) + \underset{\sim}{W}_L \ H(\theta+\theta_L-2\pi) + \underset{\sim}{W}_T \ H(\theta+\theta_T-2\pi) \tag{5}$$

where H( ) is the Heaviside step function. The relevant angles are defined by

$$c_L \cos\phi_T = c_T \cos\phi_L \tag{6}$$

$$c_L \sin\phi_T \cos\theta_T = c_T \sin\phi_L \cos\theta_L \tag{7}$$

In these relations $c_T = (\mu/\rho)^{\frac{1}{2}}$ is the velocity of transverse waves. The terms in Eq.(5) are

$$\underset{\sim}{W}_{inc} = \underset{\sim}{p} \exp[ik_L(p_1x_1+p_2x_2)] \tag{8}$$

$$\underset{\sim}{W}_L = R_L \underset{\sim}{p}^L \exp[ik_L(p_1^Lx_1+p_2^Lx_2)] \tag{9}$$

$$\underset{\sim}{W}_T = R_T \underset{\sim}{d} \exp[ik_T(p_1^Tx_1+p_2^Tx_2)] \tag{10}$$

where R_L and R_T are the usual reflection coefficients for the reflection of a plane longitudinal wave which is incident on a stress-free surface under an angle ϕ_o with the normal, and $k_T = \omega/c_T$. We have

$$R_L = \frac{\sin 2\phi_o \ \sin 2\phi_2 - (c_L/c_T)^2 \ \cos^2 2\phi_2}{\sin 2\phi_o \ \sin 2\phi_2 + (c_L/c_T)^2 \ \cos^2 2\phi_2} \tag{11}$$

$$R_T = \frac{2(c_L/c_T) \ \cos 2\phi_2 \ \sin 2\phi_o}{\sin 2\phi_o \ \sin\phi_T + (c_L/c_T)^2 \ \cos^2 2\phi_T} \tag{12}$$

The unit vectors appearing in Eqs.(8) - (10) are

$$\underset{\sim}{p}^L = (p_1^L, p_2^L, p_3^L) = (p_1, -p_2, p_3) \tag{13}$$

$$\underset{\sim}{p}^T = (\sin\phi_T \ \cos\theta_T, \ -\sin\phi_T \ \sin\theta_T, \ \cos\phi_T) \tag{14}$$

$$\underset{\sim}{d} = \underset{\sim}{p}^T \times (\underset{\sim}{i}_2 \times \underset{\sim}{p}^T) \ / \ |\underset{\sim}{i}_2 \times \underset{\sim}{p}^T| \tag{15}$$

Now let us consider an incident longitudinal wave whose surfaces of constant phase are arbitrary but smooth. The wave is incident on a stress free crack of arbitrary shape, whose edge is a smooth curve. The geometry is shown in Fig.2. We consider a point O on the edge, and we define an orthonormal coordinate system (x_1,x_2,s). Here s is arc length along the edge, with positive direction such that the propagation vector of the incident wave makes an acute angle, $\phi_L(s)$, with the tangent to the edge. The positive x_1 axis is in the crack surface, and in the direction of the principal normal to the edge. The x_2-axis is in the direction of the binormal to the edge, and such that the coordinates (x_1,x_2,s) form a right-hand coordinate system. At point O the incident wave may be represented by

$$\underset{\sim}{u}_{inc} = A(x_1,x_2,s)\ \nabla S\ \exp[ik_L S(x_1,x_2,s)] \tag{16}$$

where $S(x_1,x_2,s)$ is the phase function which defines surfaces of constant phase or wavefronts. The rays are normal to the wavefronts. At point O the direction of the ray is defined by the propagation vector $\underset{\sim}{p}(s)$, where

$$\underset{\sim}{p}(s) = \underset{\sim}{\nabla}S(0,0,s) = (\sin\phi_L \cos\theta_L,\ \sin\phi_L \sin\theta_L,\ \cos\phi_L) \tag{17}$$

Thus θ_L is the angle with the x_1 axis of the projection of $\underset{\sim}{p}$ on the (x_1,x_2)-plane. It is noted that the definition of $\underset{\sim}{p}(s)$ given by Eq.(17) is completely analogous to Eq.(3), with the understanding that all quantities now are functions of the arc length s. Considering some point $\bar{s}$ on the edge, it follows that the phase S_o at point s can be expressed as

$$S_o(s) \equiv S(0,0,s) = S(0,0,\bar{s}) + \int_{\bar{s}}^{s} \cos[\phi_L(s)]ds \tag{18}$$

For <u>small</u> x_1 and x_2, the geometrical optics part of the diffracted field corresponding to the incident field given by Eq. (16) can now readily be constructed. We find

$$\underset{\sim}{u}_{go}(x_1,x_2,s) = A_o(s)\ \exp[ik_L S_o(s)]\ \underset{\sim}{W}_{go}(x_1,x_2) + o(1) \tag{19}$$

where W_{go} is defined by Eq.(5). The o(1) term in Eq.(19) contains the effect of curvature of the wavefront at the edge. Also

$$A_o(s) = A(0,0,s) \tag{20}$$

The expression given by Eq.(19) is taken as the outer solution in a matching of two solutions.

Examination of the geometrical optics part of the displacement field, Eq.(19), motivates the following choice of the displacement in a small region near the edge (the inner solution)

$$\underset{\sim}{u}_{is}(y_1,y_2,s) = \exp[ik_L S_o(s)][\underset{\sim}{v}(y_1,y_2,s) + O(k_L^{-1})] \tag{21}$$

where the inner variables y_i are given by

$$y_i = k_L x_i \;, \; i = 1,2 \tag{22}$$

The choice of the stretching factor k_L is based on the requirement that the s-derivatives of the inner solution should be of the same order of magnitude as the y_i-derivatives.

In the coordinate system (y_1,y_2,s), the metrics are $h_1 = h_2 \approx k_L^{-1}$, and $h_3 = 1 + O(k_L^{-1})$. We remark that the curvature of the edge is contained in the $O(k_L^{-1})$ term in the metric h_3. With $j,k = 1,2$ and implying the summation convention, the elastodynamic displacement equations of motion to leading order in k_L become upon substitution of (21)

$$k_L^2\,[v_{j,kk}+(c_L/c_T)^2(v_{k,kj}+ i\, v_{3,j}\cos\phi_L + v_j) - v_j\cos^2\phi_L]+O(k_L) = 0 \tag{23}$$

$$k_L^2\,[v_{3,kk}+(c_L/c_T)^2(i\, v_{k,k}\cos\phi_L+ v_3\sin^2\phi_L) - v_3\cos^2\phi_L]+O(k_L) = 0 \tag{24}$$

On the surfaces of the crack, the conditions of vanishing surface tractions yield

$$k_L\,[\lambda\,(v_{k,k} + i\, v_3\cos\phi_L)\,\delta_{j2} + \mu\,(v_{j,2} + v_{2,j})] + O(1) = 0 \tag{25}$$

$$k_L\,[\mu\,(v_{3,2} + i\, v_2\cos\phi_L)] + O(1) = 0 \tag{26}$$

Equation (21) defines the inner solution

The matching of the inner and outer solutions is carried out on an overlap region for which $x_i = k_L^{-3/4}\epsilon_i$ and $y_i = k_L^{\frac{1}{4}}\epsilon_i$, where ϵ_i satisfy the requirement $0 < (\epsilon_1^2+\epsilon_2^2)^{\frac{1}{2}} < \infty$. The coefficients -3/4 is fairly arbitrary, except that its absolute value should be between 0.5 and 1. The matching condition is

$$\underset{\sim}{u}_{go}(k_L^{-3/4}\epsilon_1,\, k_L^{-3/4}\epsilon_2,s) - \underset{\sim}{u}_{is}(k_L^{\frac{1}{4}}\epsilon_1,k_L^{\frac{1}{4}}\epsilon_2,s) \sim o(1) \tag{27}$$

where $\underset{\sim}{u}_{go}$ and $\underset{\sim}{u}_{is}$ are defined by Eqs.(19) and (21), respectively.

Equation (27) holds in the overlap regions, except on the boundary of the shadow region, and the boundary of the region of reflected rays. We remark that the diffracted field, which is discussed in detail in Ref.[6], enters the matching condition in subsequent terms, but not in the terms of the order that are considered here.

Let us now return to the diffraction of a plane wave of the form given by Eq.(2), by a semi-infinite crack. The total displacement field for this problem depends on x_3 only through the factor $\exp(ik_L p_3 x_3)$. Thus this displacement field may be represented by an expression of the form

$$\underset{\sim}{u}(x_1,x_2,x_3) = \exp(ik_L p_3 x_3)\ \underset{\sim}{W}(x_1,x_2) \tag{28}$$

It can now be verified that

$$\underset{\sim}{v}(y_1,y_2,s) = A_o(s)\ \underset{\sim}{W}(y_1/k_L, y_2/k_L) \tag{29}$$

satisfies the displacement equations of motion (23) and (24) as well as the boundary conditions (25) and (26) exactly to the order indicated. Moreover, with $A_o(s)$ defined by Eq.(20), it is found that the matching condition (27) is satisfied to the order indicated, since for $y_i = k_L^{\frac{1}{4}}\epsilon_i$, $\underset{\sim}{W}(k_L^{-3/4}\epsilon_1, k_L^{-3/4}\epsilon_2)$ asymptotically equals $\underset{\sim}{W}_{go}(k_L^{-3/4}\epsilon_1, k_L^{-3/4}\epsilon_2)$. On the basis of these observations, Eq.(21) then yields for the inner solution

$$\underset{\sim}{u}_{is}(x_1,x_2,s) \sim A_o(s)\ \exp[ik_L S_o(s)]\ \underset{\sim}{W}(x_1,x_2) + O(k_L^{-1}) \tag{30}$$

This result implies that the stress intensity factor at a point s on the edge is equal to the field of the incident ray at this point, times the stress intensity factor divided by the phase $\exp(ik_L p_3 x_3)$ for a plane wave of unit amplitude which is incident with the same propagation vector p.

3. Elastodynamic Stress Intensity Factors

Elastodynamic stress intensity factors for a straight-edged semi-infinite crack were analyzed by Achenbach and Gautesen [7]. The diffraction of a plane longitudinal wave of the form

$$\underset{\sim}{u}^{(1)} = \underset{\sim}{p} \exp(ik_L \underset{\sim}{p} \cdot \underset{\sim}{x}) \tag{31}$$

which is incident under an arbitrary angle with the edge of the crack, generates Mode I, II and III stress intensity factors. An incident transverse wave of the form

$$\underset{\sim}{u}^{(m)} = \underset{\sim}{d}^{(m)} \exp(ik_T \underset{\sim}{p} \cdot \underset{\sim}{x}) \tag{32}$$

generally also generates stress intensity factors in all three fracture modes. In Eq. (32) the index m can assume the values m = 2 and m = 3, which refer, respectively, to vertical and horizontal polarization of the incident wave, relative to the plane spanned by $\underset{\sim}{i}_2$ and $\underset{\sim}{p}$, see Fig.1. Thus,

$$\underset{\sim}{d}^{(2)} = \underset{\sim}{d}^{(3)} \times \underset{\sim}{p} \tag{33}$$

$$\underset{\sim}{d}^{(3)} = \underset{\sim}{p} \times \underset{\sim}{i}_2 \,/\, |\, \underset{\sim}{p} \times \underset{\sim}{i}_2 \,| \tag{34}$$

For each incident field $\underset{\sim}{u}^{(1)}$ and $\underset{\sim}{u}^{(m)}$, m = 2,3, the three stress intensity factors may be represented by a vector $\underset{\sim}{K}_o^{(m)}$. For an incident longitudinal wave the components of $\underset{\sim}{K}_o^{(1)}$ are presented in Ref.[7]. The stress intensity factors generated by incident transverse waves can be computed by the methods of Ref.[7]. For convenience, the components of

$$\underset{\sim}{K}^{(1)} = \underset{\sim}{K}_o^{(1)} \exp(-ik_L p_3 x_3) \tag{35}$$

$$\underset{\sim}{K}^{(m)} = \underset{\sim}{K}_o^{(m)} \exp(-ik_T p_3 x_3) \quad , \qquad m = 2,3 \tag{36}$$

are listed in Appendix A. Here $\underset{\sim}{K}_1^{(m)}$, $\underset{\sim}{K}_2^{(m)}$, $\underset{\sim}{K}_3^{(m)}$ are the Mode I, II and III stress intensity factors, respectively.

In this paper we are interested in the stress intensity factors generated at the edge of a curved crack, see Fig.2 by an incident wave of the form

$$\underset{\sim}{u} = A(x_1,x_2,s)\, \underset{\sim}{e}^{(m)}(s) \exp[ik^{(m)} S(x_1,x_2,s)] \tag{37}$$

where m = 1 signifies a longitudinal wave, when

$$\underset{\sim}{e}^{(1)} \equiv \underset{\sim}{p} = \underset{\sim}{\nabla} S \;, \qquad k^{(1)} = k_L \tag{38}$$

while m = 2 signifies a transverse wave with vertical polarization

$$\underset{\sim}{e}^{(2)} = \underset{\sim}{d}^{(2)}, \quad k^{(2)} = k_T \tag{39}$$

and m = 3 refers to a transverse wave with horizontal polarization

$$\underset{\sim}{e}^{(3)} = \underset{\sim}{d}^{(3)}, \quad k^{(3)} = k_T \tag{40}$$

It was shown in the previous section that for an incident wave of arbitrary shape the stress intensity factor at a point s on a crack edge is equal to the field of the incident ray at this point multiplied by the stress intensity factor for a corresponding plane wave divided by the phase $\exp(ik^{(m)}p_3x_3)$. Thus

$$\underset{\sim}{K} = \mathrm{K}\, A_o\, \underset{\sim}{e}_o^{(m)} \exp(ik^{(m)}S_o) \tag{41}$$

where $A_o \underset{\sim}{e}_o^{(m)} \exp(ik^{(m)}S_o)$ is the field of the incident ray at a point on the edge, and K is a matrix, whose components are

$$\mathrm{K}_{jk} = K_j^{(1)}p_k + K_j^{(2)}d_k^{(2)} + K_j^{(3)}d_k^{(3)} \tag{42}$$

The coefficients $K_j^{(m)}$ are defined by Eqs.(35) and (36).

Several specific examples of elastodynamic stress intensity factors generated by body waves are discussed in Section 5.

4. Surface Waves

The results of the preceding sections can easily be extended to the computation of elastodynamic stress intensity factors which are generated at the diffraction of Rayleigh surface waves by a crack edge. In this paper we consider stress intensity factors due to surface waves which are incident on an edge via both faces of a crack. Then, it is convenient to distinguish between symmetric and antisymmetric motions relative to the plane of the crack. Only the case of symmetric motions is considered in detail, but the corresponding results for antisymmetric motions can be obtained analogously.

Following Eqs.(3.12) and (3.13) of Ref.[8] with appropriate minor adjustments in notation, the time-reduced displacements for the incident surface wave may be written in the forms

$$u_2^R = [(1-2s_R^2/s_T^2)\, e^{-\omega a|x_2|} + 2(s_R^2/s_T^2)\, e^{-\omega b|x_2|}]\, V_2 \mathrm{sgn}(x_2) \tag{43}$$

$$u_j^R = \frac{1}{\omega a}[(2s_R^2-s_T^2)\, e^{-\omega a|x_2|} - 2(ab/s_T^2)\, e^{-\omega b|x_2|}]\, \frac{\partial V_2}{\partial x_j} \tag{44}$$

where j = 1,3, and

$$V_2 = \exp\{i\omega[-s_R(x_1\sin\theta_i + x_3\cos\theta_i)]\} \tag{45}$$

$$a = (s_R^2-s_L^2)^{\frac{1}{2}} \quad , \quad b = (s_R^2-s_T^2)^{\frac{1}{2}} \tag{46a,b}$$

Thus, in the geometry shown in Fig.1, the surface wave is incident on the edge under an angle θ_i with the edge.

The diffraction of surface waves has been discussed in some detail in Refs.[8] and [9]. If we denote the time reduced displacement in the x_2-direction on the faces of the crack by $U_2^+(x_1,x_2)$, the exponential Fourier transform with respect to x_1 of U_2^+ is given by Eq. (44) of Ref.[8]. Here we slightly rewrite this expression in the form

$$\bar{U}_2^+(\xi) = -\frac{1}{i\omega}\left\{\frac{1}{\xi-s_R\sin\theta_i+i0} - \frac{1}{\xi+s_R\sin\theta_i+i0}\right\}\frac{E(s_R\sin\theta_i)}{E(\xi)}\, e^{-i\omega\eta x_3} \tag{47}$$

In Eq.(47)

$$\eta = s_R\cos\theta_i \tag{48}$$

$$E(\xi) = K^+(\xi,\eta)/[(s_L^2-\eta^2)^{\frac{1}{2}}+\xi]^{\frac{1}{2}} \tag{49}$$

where $K^+(\xi,\eta)$ is defined by Eq.(4.6) of Ref.[8]. The incident and reflected surface waves correspond to the contributions of the poles in inversion integrals. Thus, the contribution at $\xi = s_R\sin\theta_i - i0$ just reproduces the incident wave given by Eq.(43), while the pole at $\xi = -s_R\sin\theta_i - i0$ gives the reflected surface wave. It is easily verified that the reflection coefficient can be written as

$$R_R = -E(s_R \sin\theta_i)/E(-s_R \sin\theta_i) \tag{50}$$

By virtue of asymptotically valid relations between a Fourier transform as $|\xi\omega| \to \infty$ and the inverse transform as $x_1 \to 0$, the following expression for the Mode I stress intensity factor has been derived as Eq.(66) of Ref.[7]:

$$K_o^R = 2^{3/2}\omega^{3/2}\mu(1-s_L^2/s_T^2)\, e^{i\pi/4} \lim_{\xi \to \infty} \bar{U}_2^{+}(\xi) \tag{51}$$

By substituting Eq.(47) into Eq.(51), we obtain $K_o^R = K^R \exp(-i\omega\eta x_3)$, where

$$K^R = -2^{5/2}\omega^{\frac{1}{2}}\mu\,(1-s_L^2/s_T^2)\, s_R \sin\theta_i\, E(s_R \sin\theta_i)\, e^{-i\pi/4} \tag{52}$$

An extension to incident surface waves with curved wavefronts, which are incident on curved crack edges, is now completely analogous to the case of body waves. Suppose the time reduced incident field is of the form

$$u_2^R = \pm A(x_1,s)\, \exp[-i\omega s_R\, S(x_1,s)] \tag{53}$$

where the displacement is positive on the upper face. The result then simply states that the stress intensity factor at a point on the edge is

$$K_I = A_o K^R \exp[-i\omega\, s_R S_o] \tag{54}$$

where

$$A_o = A(0,s) \quad , \quad S_o = S(0,s) \tag{55a,b}$$

Surface waves of the type given by Eq.(53) can be generated by direct application of disturbances to the faces of the crack, as well as by diffraction processes. Examples of both cases are given in the next section.

5. Examples

In this section several applications of Eqs.(41) and (54) are presented. We will consider examples with curved as well as with plane wavefronts, and with curved as well as with straight crack edges. Both steady-state time harmonic and impact excitations will be considered.

Anti-Plane Motions

The first example is concerned with the stress intensity factor at the tip of a semi-infinite crack due to an anti-plane line load of P force units per unit length and time variation exp(-iωt), which is applied parallel to the crack as shown in Fig.3. This elastodynamic problem is two-dimensional, and the time reduced anti-plane displacement $u_3(x_1,x_2)$ is governed by

$$\nabla^2 u_3 + k_T^2 u_3 = -(P/\mu)\,\delta(\underset{\sim}{r} - \underset{\sim}{r}_P) \tag{56}$$

The solution to Eq.(56) is

$$u_3(r,r_p) = \frac{i}{4}\frac{P}{\mu} H_o^{(1)}(k_T|\underset{\sim}{r} - \underset{\sim}{r}_P|) \tag{57}$$

For large frequencies we may write

$$u_3(r,r_P) \sim \frac{i}{4}\frac{P}{\mu}\left(\frac{2}{\pi k_T}\frac{1}{|\underset{\sim}{r}-\underset{\sim}{r}_P|}\right)^{\frac{1}{2}} \exp\left[ik_T|\underset{\sim}{r}-\underset{\sim}{r}_P| - \frac{i\pi}{4}\right] \tag{58}$$

Equation (58) represents the incident field.

For an incident plane horizontally polarized transverse wave of unit amplitude, the elastodynamic stress intensity factor was presented by Eq.(90) of Ref.[7] as

$$\left(K_o^{(3)}\right)_3 = 2\mu k_T^{\frac{1}{2}} e^{-i\pi/4} \sin(\theta_T/2) \tag{59}$$

where θ_T is the angle of incidence. For the problem at hand we have $p_1 = \cos\theta_T = [(x_1)_0 - (x_1)_P]/|\underset{\sim}{r}_o - \underset{\sim}{r}_P|$. Equation (59) can then be rewritten as

$$\left(K_o^{(3)}\right)_3 = \mu(2k_T)^{\frac{1}{2}} (1 - p_1)^{\frac{1}{2}} e^{-i\pi/4} \tag{60}$$

Equation (41), together with Eqs.(58) and (60) then yields for the line load problem

$$K_3 = \frac{1}{2} P \left(\frac{1}{\pi} \frac{1}{|\underset{\sim}{r}_o - \underset{\sim}{r}_p|}\right)^{\frac{1}{2}} (1-p_1)^{\frac{1}{2}} \exp[ik_T|\underset{\sim}{r}_o - \underset{\sim}{r}_p|] \tag{61}$$

This result agrees with the near-tip field computed from the asymptotic expansion of the exact solution of an analogous problem, see Ref.[10], p. 329, Eq.(8.74).

For the second example we consider a crack of finite length 2ℓ, subjected to a plane horizontally polarized transverse wave of the form

$$(u_3)_{inc} = A \exp[i\omega(s_T x_1 \cos\theta_T + s_T x_2 \sin\theta_T - t)] \tag{62}$$

The geometry is shown in Fig.4. To compute the Mode III elastodynamic stress intensity factor at high values of $\omega s_T \ell$, we trace an incident ray as it undergoes diffractions at the two crack tips. The primary diffraction produces diffracted rays on the faces of the crack, whose fields for large frequency and/or distance are easily computed, as shown in Ref.[6]. These fields are subsequently diffracted at the other crack tip, thus producing another set of rays on the crack faces. This process continues, and one can visualize systems of rays travelling back and forth between the two crack tips. Since the fields decay as $(\omega s_T \ell)^{-\frac{1}{2}}$, it is only necessary to consider a few secondary diffractions. In fact, in the computations presented here, only one secondary diffraction is included.

The primary diffracted field of the incident ray at 0_1, see Fig.4, is the diffracted field for a semi-infinite crack. This field was discussed in Ref.[6]. On the upper surface of the crack (i.e., at $\theta = 0$)

the exponential Fourier transform of the anti-plane displacement is given by Eq.(89) of Ref.[7] as

$$\bar{U}_3^+ = (2k_T)^{\frac{1}{2}} A \left[i\omega^{3/2}(\xi+s_T\cos\theta_T)(\xi+s_T)^{\frac{1}{2}} \right]^{-1} \sin(\theta_T/2) \tag{63}$$

Analogously to the results given by Eqs.(65),(70) and (71) of Ref.[6], the far field (or high frequency) inversion at $\theta = 0$ can be computed as

$$U_3^+ \approx (x_1)^{-\frac{1}{2}} D_T^+ A e^{i\omega(s_T x_1 - t)} \tag{64}$$

where the diffraction coefficient D_T^+ is

$$D_T^+ \approx \left(\frac{1}{2\pi\omega s_T}\right)^{\frac{1}{2}} \frac{1}{\sin(\theta_T/2)} e^{i\pi/4} \quad , \quad 0 < \theta_T < \pi \tag{65}$$

The primary diffracted field at 0_2 is obtained by taking into account that the angle of incidence measured from the shadow side of the crack is $\pi-\theta_T$, as shown in Fig.4. For the primary diffraction we can then write

$$U_3^- = (-x_1+2\ell)^{-\frac{1}{2}} D_T^- A e^{i\omega(-s_T x_1 + 2s_T\ell - t)} \tag{66}$$

where

$$D_T^- = \left(\frac{1}{2\pi\omega s_T}\right)^{\frac{1}{2}} \frac{1}{\cos(\theta_T/2)} e^{i\pi/4} \quad , \quad 0 < \theta_T < \pi \tag{67}$$

The corresponding displacements on the lower crack faces are equal in magnitude but opposite in sign to Eqs.(64) and (66).

For the primary incident ray the stress intensity factor at $x_1 = 0$ is given by Eq.(59), which is valid in the range $0 \le \theta_T \le \pi$. For $\theta_T = \pi$, the waves approach the crack tip from the side of the crack face. Then, not only the incident wave but also the reflected wave is incident on the crack tip. For this system of incident and reflected waves, the displacements on the crack face are actually twice as large as would correspond to the incident wave only. This observation implies that for an incident wave which is given as a surface displacement on the lower face of the crack, the elastodynamic stress intensity factor is half the value given by Eq.(59) for $\theta_T = \pi$.

On the basis of the foregoing observation, the stress intensity factor at $x_1 = 0$ due to the primary diffracted wave generated on the lower crack face (which is minus the expression given by Eq.(66)),then follows by setting $\theta_T = \pi$ in Eq.(59) and substituting the appropriate amplitude, as

$$K_{III} = - \mu k_T^{\frac{1}{2}} e^{-i\pi/4}(2\ell)^{-\frac{1}{2}} D_T^- A e^{2i\omega s_T \ell} \tag{68}$$

Multiplying this result by 2, to account for the diffracted wave on the upper face, which is given by Eq.(66), and adding the result to the primary stress intensity factor given by Eq.(59), we obtain at $x_1 = 0$

$$|K_{III}| \sim 2\mu A k_T^{\frac{1}{2}} |e^{-i\pi/4}[\sin(\theta_T/2) - (2\ell)^{-\frac{1}{2}} D_T^- e^{2i\omega s_T \ell}]| \tag{68}$$

It is easily shown that the incident displacement wave given by Eq.(62) corresponds to a stress wave of magnitude τ_o, where $\tau_o = \mu A k_T$. Let us now consider the special case of normal incidence, when $\theta_T = \pi/2$. Then we have

$$|K_{III}/\tau_o \ell^{\frac{1}{2}}| \sim \left(\frac{2}{k_T \ell}\right)^{\frac{1}{2}} |e^{-i\pi/4}[1-\left(\frac{1}{\pi k_T \ell}\right)^{\frac{1}{2}} e^{i(2k_T \ell + \pi/4)}]| \tag{69}$$

which can be further simplified to

$$|K_{III}/\tau_o \ell^{\frac{1}{2}}| \sim \left(\frac{2}{k_T \ell}\right)^{\frac{1}{2}} [1 - \left(\frac{1}{\pi k_T \ell}\right)^{\frac{1}{2}} \cos(2k_T \ell + \pi/4)] \tag{70}$$

This stress intensity factor was computed by other methods by Mal [11]. The results are compared in Fig.5. It is noted that the agreement is excellent for $k_T \ell > 1.5$.

In-Plane Motions

As an example of the much more complicated class of in-plane problems we will consider the elastodynamic stress intensity factor generated by the diffraction of a longitudinal wave by a crack of length 2ℓ.

Similar to the anti-plane problem the elastodynamic stress intensity factor is computed by tracing rays as they are diffracted by the two crack tips. There are, however, some interesting and important differences with the case of anti-plane motions. The most important difference is related to the significance of Rayleigh surface waves in the primary and secondary diffractions. The primary diffraction generates not only rays of longitudinal and transverse motion, but also surface motions near the faces of the crack. It is a general feature of in-plane problems that bulk waves decay as $(\text{distance})^{-\frac{1}{2}}$, while surface waves do not decay at all. Thus, on the faces of the crack, the surface motions tend to predominate the longitudinal and transverse motions due to the body waves. This means that it is necessary to consider stress intensity factors due to surface waves, which were discussed in Section 4. Since there is no spatial decay, except for the fact that the reflection coefficient R_R given by Eq.(50) is smaller than unity, it is necessary to sum the influence of all secondary diffractions of the surfaces waves. Here we present the details for the case of normal incidence.

The Rayleigh surface wave generated by the primary diffraction of the incident ray follows from Eq.(46) of Ref.[6]. For the problem at hand we have

$$(u_2)_{inc} = A\,\exp[i\omega(s_L x_2 - t)] \tag{71}$$

Thus $p_1 = p_3 \equiv 0$, $p_2 = 1$. The exponential Fourier transform of the surface displacement for $x_1 \geq 0$ then follows from Eq.(46) of Ref.[6] as

$$\bar{U}_2^+ = - \frac{i s_T^2/s_L^{\frac{1}{2}}}{2^{\frac{1}{2}}\omega(s_T^2 - s_L^2)^{\frac{1}{2}}} \frac{1}{\xi} \frac{(s_L+\xi)^{\frac{1}{2}}}{s_R + \xi} \frac{A}{K^+(\xi)} \tag{72}$$

The surface wave contribution comes from the pole at $\xi = - s_R$.

We find

$$u_2^R = D_R\, A\, e^{i\omega(s_R x_1 - \omega t)} \tag{73}$$

where

$$D_R = - i \frac{s_T}{s_R} \left[\frac{s_R/s_L - 1}{2(1-s_L^2/s_T^2)}\right]^{\frac{1}{2}} \frac{1}{K^+(-s_R)} \tag{74}$$

The Mode I stress intensity factor due to the primary diffraction immediately follows from Eq.(41) by setting $p_2 = p_3 = 0$, $p_1 = 1$. We find

$$K_I^B = K_1^{(1)}\, A \tag{75}$$

where $K_1^{(1)}$ can be obtained either as a special case of Eq.(A.2), or by employing Eq.(51) together with Eq.(72). The result is

$$K_1^{(1)} = 2\mu A(s_T^2/s_L^2 - 1)^{\frac{1}{2}}(\omega s_L)^{\frac{1}{2}}\, e^{-i\pi/4} \tag{76}$$

The secondary diffractions at point 0_1 are produced by Rayleigh surface waves. The phases of subsequent contributions as a ray travels back and forth between the crack tips increase with $2\omega s_R \ell$ with each diffraction, i.e., they are $\exp(2i\omega s_R \ell)$, $\exp(4i\omega s_R \ell)$, etc. The magnitudes of subsequent contributions are reduced by multiplication by the reflection factor R_R given by Eq.(50). Thus, for the contributions of the Rayleigh waves to the stress intensity factors, we have

$K_I^R = K^R A$, where

$$K^R = D_R K_o^R [e^{2i\omega s_R \ell} + R_R e^{4i\omega s_R \ell} + R_R^2 e^{6i\omega s_R \ell} + \cdots] \tag{77}$$

where K_o^R is defined by Eq.(52). Equation can be written in closed form as

$$K^R = \frac{D_R K_o^R \exp(2i\omega s_R \ell)}{1-R_R \exp(2i\omega s_R \ell)} \tag{78}$$

The total stress intensity factor is

$$K_I = K_I^B + K_I^R \tag{79}$$

Numerical results were worked out for $s_T^2/s_L^2 = 3$, which corresponds to a material with a Poisson's retain $\nu = 0.25$. For that case we have $K^+(s_R) = 0.91116$ and $K^+(-s_R) = 1.9009$. The absolute value of K_I is compared in Fig.5 with numerical results obtained by Mal [11]. It is noted that the agreement is very good for $k_T \ell > 1.5$.

Normal Point Loads on a Semi-Infinite Crack

As an example of the computation of elastodynamic stress intensity factors generated by surface waves only, we consider the case that the faces of a semi-infinite crack are subjected to equal and opposite time-harmonic point loads applied at $x_1 = x_o$, $x_2 = \pm 0$, and $x_3 = 0$, see Fig.6. The boundary conditions at $x_2 = 0$ then are

$$\tau_{22} = - N \,\delta(x_1 - x_o)\,\delta(x_3) \exp(-i\omega t) \tag{80}$$

$$\tau_{21} = \tau_{23} = 0 \tag{81}$$

Motions generated by a normal point load on a half-space have been studied in detail. It is well known that for $\omega s_R r >> 1$, where $r^2 = (x_1 - x_o)^2 + x_3^2$, the predominant component of the surface displacement is the Rayleigh surface wave. The normal surface displacement is given by Eq.(2-120) of Ref.[12]. Taking into account that in the present paper we use a negative time exponent, $\exp(-i\omega t)$, the time-reduced displacement can be written as

$$U_2 \sim A(r)\, e^{i(\omega s_L r + \pi/4)} \tag{82}$$

where

$$A(r) = -\frac{\omega^{\frac{1}{2}} s_R^{\frac{1}{2}} N}{\mu}\left(\frac{1}{2\pi r}\right)^{\frac{1}{2}} \frac{s_T^2 (s_R^2 - s_L^2)^{\frac{1}{2}}}{R'(s_R)} \tag{83}$$

In Eq.(83), $R(\zeta)$ is the Rayleigh function

$$R(\zeta) = (2\zeta^2 - s_T^2)^2 + 4\zeta^2 (s_L^2 - \zeta^2)^{\frac{1}{2}} (s_T^2 - \zeta^2)^{\frac{1}{2}} \tag{84}$$

and the prime denotes the derivative with respect to ζ.

In the context of the discussion of Section 4, and referring to Fig. 6, we can now define at a point x_3 on the edge

$$\sin\theta_i = x_o/\bar{r}\ ;\ \cos\theta_i = x_3/\bar{r}\ ;\ \bar{r} = (x_o^2 + x_3^2)^{\frac{1}{2}} \tag{85}$$

By comparing Eqs. (53) and (82) we can now immediately identify $A(x_1,s)$ and $S(x_1,s)$. By virtue of Eq. (54) the stress intensity factor may then be written

$$K_I = A(\bar{r})\, K_o^R \exp[i\omega(s_R \bar{r} + \pi/4)] \tag{86}$$

This stress intensity factor was also computed in Ref.[7] by a direct asymptotic evaluation of integrals in an expression for the exact solution. It can be shown that Eq. (86) agrees with Eq.(104) of Ref.[7].

Penny-Shaped Cracks

For the final example we consider a penny-shaped crack under the influence of an axially symmetric torsional wave. In cylindrical coordinates (r,θ,x_2), axially symmetric torsional wave motions are defined by a single displacement component $u_\theta(r,\theta,x_2)$ which is independent of the angular variable θ. The only non-zero stress components are $\tau_{2\theta}$ and $\tau_{r\theta}$. In the problem considered here the axis of symmetry is normal to the plane of the crack, and passes through the center of the penny-shaped crack. The geometry is shown in Fig.7. The incident stress wave is of the form

$$\tau_{2\theta} = \mu \frac{\partial u_\theta}{\partial x_3} = \frac{r\tau_o}{a} H(c_T t - x_2) \tag{87}$$

Thus, this is a transient stress wave, which strikes the crack at time $t = 0$. Writing the near-tip stress field in the form

$$\tau_{2\theta} = \frac{1}{(2\pi)^{\frac{1}{2}}} \frac{K_3(t)}{(r-a)^{\frac{1}{2}}} \tag{88}$$

the first term in an expansion over time follows from the results of Kennedy and Achenbach [13, Eq.(3.11)] as

$$K_3(t) = 2\tau_o (2c_T t/\pi)^{\frac{1}{2}} [1 + O(t)] \tag{89}$$

Now we will approach this problem with the theory presented in this paper. The incident wave $(u_\theta)_{inc}$ can be expressed as

$$(u_\theta)_{inc} = -\frac{c_T \tau_o r}{2\pi \mu a} \int_{-\infty}^{\infty} \frac{1}{\omega^2} \exp[i\omega(x_2/c_T - t)] d\omega \tag{90}$$

Treating the quantity $(c_T t_o r/2\pi \mu a\omega^2)\exp(i\omega x_2/c_T)$ as the incident wave, it is observed that this wave is equivalent to an incident plane horizontally polarized time-harmonic wave, which is normally incident on the crack, i.e., $p_1 = 0$. It then follows by employing Eqs.(41) and (60) that

$$K_3 = \mu \left(\frac{2\omega}{c_T}\right)^{\frac{1}{2}} e^{-i\pi/4} \frac{c_T \tau_o}{2\pi \mu \omega^2} \tag{91}$$

Now we can introduce the time factor $\exp(-i\omega t)$ in Eq.(91) and substitute the result in the Fourier superposition integral. Since

$$\int_{-\infty}^{\infty} \omega^{-3/2} e^{-i\omega t} \, dt = 4(\pi t)^{\frac{1}{2}} e^{i\pi/4} \tag{92}$$

the evaluation of the superposition integral yields Eq.(89), and the results obtained by the method of this paper agree with those of Ref.[13]. This example also confirms the expected result that the large frequency approximation in the frequency domain leads to a small time approximation in the time domain. In this context small time means small after arrival of the wavefront which separates the disturbed and the undisturbed regions of the solid.

ACKNOWLEDGEMENT

This work was carried out in the course of research sponsored by the Office of Naval Research under Contract ONR N00014-76-C-0063.

References

1. Friedlander, F. G., Sound Pulses, Cambridge, University Press, 1958.

2. Karal, F. C., Jr., and Keller, J. B., "Elastic Wave Propagation in Homogeneous and Inhomogeneous Media," J. Acoust. Soc. Am. 31, 1959, p. 694.

3. Keller, H. B., "Propagation of Stress Discontinuaties in Inhomogeneous Elastic Media," SIAM Rev. 6, 1964, p. 356.

4. Keller, J B., "A Geometrical Theory of Diffraction," in Calculus of Variations and its Applications, McGraw-Hill, New York, 1958.

5. Buchal, R. N., and Keller, J. B., "Boundary Layer Problems in Diffraction Theory," Comm. Pure Appl. Mathematics 13, 1960, p. 85.

6. Achenbach, J. D., and Gautesen, A. K., "Geometrical Theory of Diffraction for Three-D Elastodynamics," J. Acoust. Sol. Am. 61, 1977, p. 413.

7. Achenbach, J. D., and Gautesen, A. K., "Elastodynamic Stress-Intensity Factors for a Semi-Infinite Crack under 3-D Loading," J. Appl. Mech. in press.

8. Achenbach, J. D., and Gautesen, A. K., "Elastic Surface Waves Guided by the Edge of a Slit," J. of Sound and Vibration, forthcoming.

9. Freund, L. B., "The Oblique Reflection of a Rayleigh Wave from a Crack Tip," Int. J. Solids and Structures 7, 1971, p. 1199.

10. Bowman, J. J., Senior, T.B.A., and Uslenghi, D.L.E., Electromagnetic and Acoustic Scattering by Simple Shapes, North-Holland, Amsterdam, 1969.

11. Mal, A. K., "Interaction of Elastic Waves with a Griffith Crack," Int. J. Eng. Sc. 8, 1970, p. 769.

12. Ewing, W. E., Jardetzky, W. S., and Press, F., Elastic Waves in Layered Media, McGraw-Hill Book Co., New York, 1957.

13. Kennedy, T. C., and Achenbach, J. D., "Rapid Extension of a Penny-Shaped Crack under Torsion," Int. J. Solids and Structures, 10, p. 519.

APPENDIX A

ELASTODYNAMIC STRESS INTENSITY FACTORS FOR THE INCIDENCE OF PLANE WAVES ON A SEMI-INFINITE CRACK

In this Appendix we have listed the components of $\underset{\sim}{K}^{(1)}$ and $\underset{\sim}{K}^{(m)}$, $m = 2,3$, which are defined by Eqs. (35) and (36). For economy of presentation the slownesses s_n, $n = 1,2,3$, are introduced as

$$s_1 = 1/c_L\ ,\quad s_2 = 1/c_T \text{ and } s_3 = 1/c_R\ , \tag{A.1a,b,c}$$

where c_R is the velocity of Rayleigh waves. In the following expressions the index (j) can assume the values (1) (incident longitudinal wave) and (2) (incident vertically polarized transverse wave). The index (3) refers to an incident horizontally polarized transverse wave. The stress intensity factors are:

$$\text{Mode I:}\qquad K_1^{(j)} = D_{1j}Q_{1j} \tag{A.2}$$

$$\text{Mode II:}\qquad K_2^{(j)} = D_{2j}Q_{2j}\left[1 + s_2Q_{3j}e^{-i\phi_{2j}}\right] \tag{A.3}$$

$$\text{Mode III:}\qquad K_3^{(j)} = is_3D_{2j}Q_{2j}Q_{3j}C_{2j}\, e^{-i\phi_{3j}} \tag{A.4}$$

$$\text{Mode I:}\qquad K_1^{(3)} = 0 \tag{A.5}$$

$$\text{Mode II:}\qquad K_2^{(3)} = s_2Q_4Q_5\, e^{-i\phi_{22}} \tag{A.6}$$

$$\text{Mode III:}\qquad K_3^{(3)} = Q_4\left[1 + is_3Q_5C_{22}\, e^{-i\phi_{32}}\right] \tag{A.7}$$

In these expressions:

$$Q_{1j} = \mu s_j\left(2\omega s_1\right)^{\frac{1}{2}} e^{-i\pi/4}\left(\sin\phi_{1j}+s_jp_1/s_1\right)^{\frac{1}{2}}\left(s_3\sin\phi_{3j}+s_jp_1\right)^{-1}/C_{1j} \tag{A.8}$$

$$Q_{2j} = \frac{-\mu s_j\left(2\omega s_2\right)^{\frac{1}{2}} e^{-i\pi/4}\left(1-p_2^2\right)\left(\sin\phi_{2j}+s_jp_1/s_2\right)^{\frac{1}{2}}}{C_{1j}\left(p_3+ip_1\right)\left(s_3\sin\phi_{3j}+s_jp_1\right)} \tag{A.9}$$

$$Q_{3j} = -\frac{2is_2p_3}{\left(p_1+ip_3\right)}\, e^{-i\phi_{2j}}\left[s_2^2e^{-2i\phi_{2j}} + 2s_3^2\left(1-s_1^2/s_2^2\right)C_{2j}^2e^{-2i\phi_{3j}}\right]$$

$$Q_4 = \mu\left(2s_2\omega\right)^{\frac{1}{2}} p_2\left(1-p_2^2\right)^{\frac{1}{2}} e^{i\pi/4}\left(p_3+ip_1\right)^{-1}\left(\sin\phi_{22}+p_1\right)^{-\frac{1}{2}} \tag{A.11}$$

$$Q_5 = 2i\left(s_3/s_2\right)\left(1-s_1^2/s_2^2\right) C_{22}Q_{32} \exp\left[i\left(\phi_{22}-\phi_{32}\right)\right] \tag{A.12}$$

$$D_{11} = \left(s_2^2/s_1^2 - 2 + 2p_2^2\right) \ ; \quad D_{21} = 2p_2 \tag{A.13a,b}$$

$$D_{12} = 2p_2\left(1 - p_2^2\right)^{\frac{1}{2}} \ ; \quad D_{22} = \left(1-2p_2^2\right)\left(1-p_2^2\right)^{\frac{1}{2}} \tag{A.14a,b}$$

$$\cos\phi_{kj} = s_j p_3/s_k \ , \quad k = 1,2,3 \tag{A.15}$$

$$C_{kj} = K^+\left(\xi_{jk} \ , \ -s_j p_3\right) \ , \quad k = 1,2 \tag{A.16}$$

where $\xi_{j1} = s_j p_1$, $\xi_{j2} = is_j p_3$

$$K^+ (\xi,\eta) = \exp\left[f(\xi,\eta)/\pi\right] \tag{A.17}$$

$$f(\xi,\eta) = \int_{q_1}^{q_2} \arctan\left[F(t,\eta)\right] (t + \xi)^{-1}dt \tag{A.18}$$

$$F(t,\eta) = \left(\eta^2+t^2\right)\left(\eta^2+t^2-s_1^2\right)^{\frac{1}{2}} \left(s_2^2-\eta^2-t^2\right)^{\frac{1}{2}} \left(\tfrac{1}{2}s_2^2-\eta^2-t^2\right)^{-2} \tag{A.19}$$

$$q_j = \left(s_j^2-\eta^2\right)^{\frac{1}{2}} \tag{A.20}$$

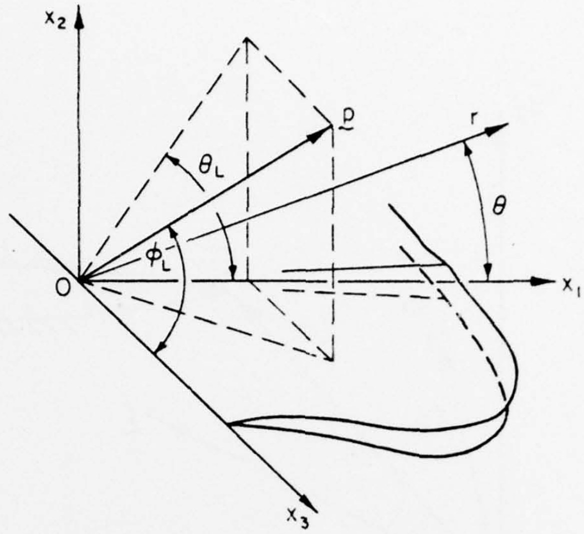

Fig. 1 Propagation vector p of plane wave incident on a crack with a straight edge

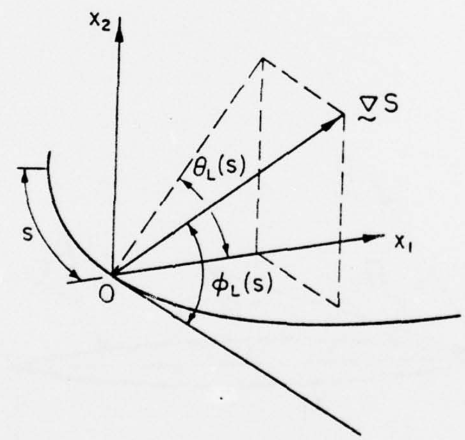

Fig. 2 Local coordinate system for wave incident on a crack with a curved edge

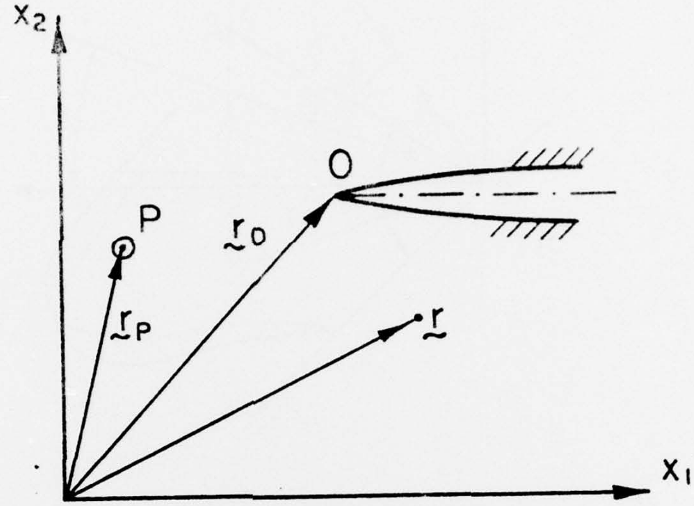

Fig. 3 Time-harmonic anti-plane line load and semi-infinite crack

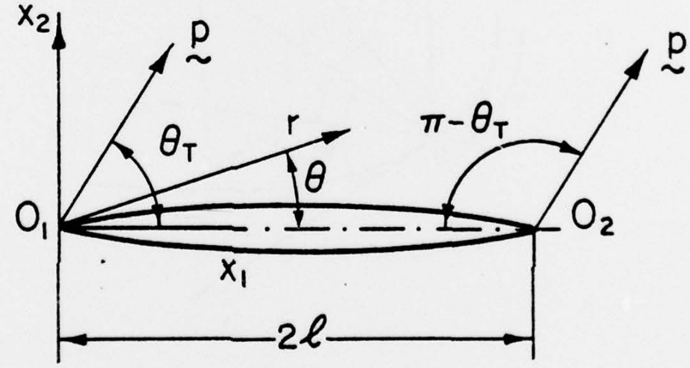

Fig. 4 Plane horizontally polarized transverse wave incident on a crack of length 2ℓ

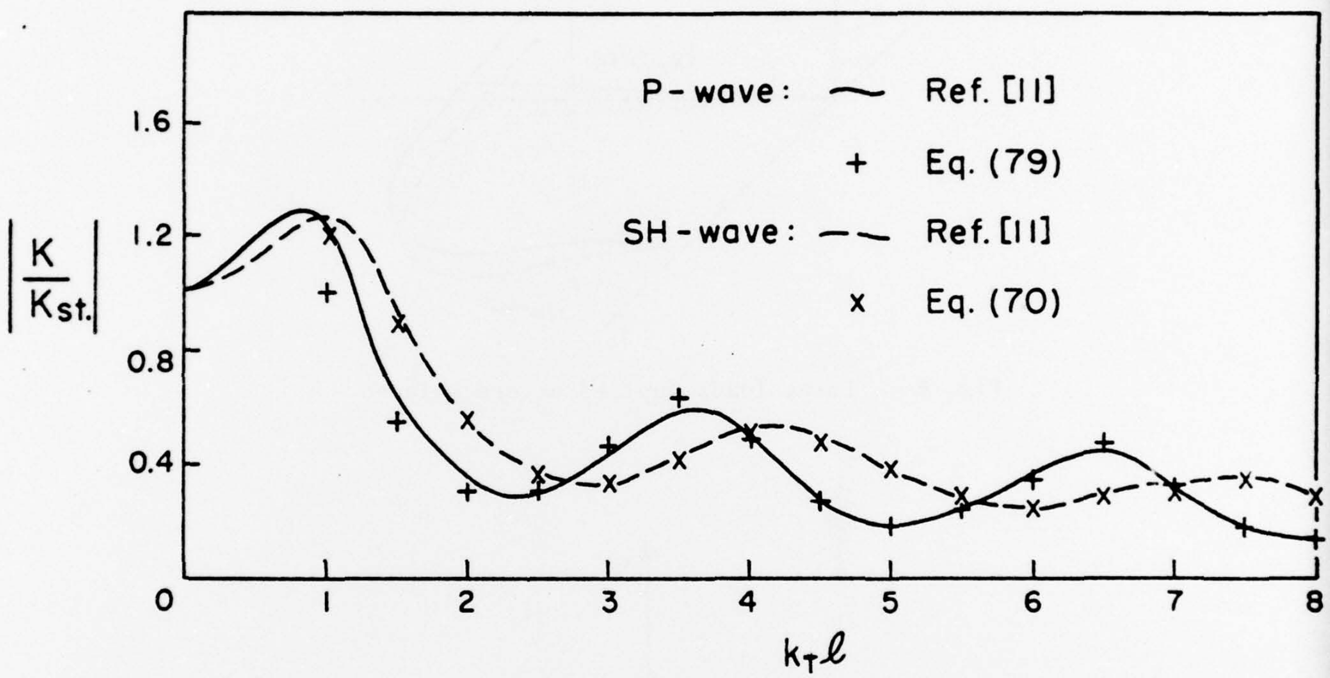

Fig. 5 Ratios of elastodynamic and elastostatic stress intensity factors versus 2π crack length/wavelength for normal incidence

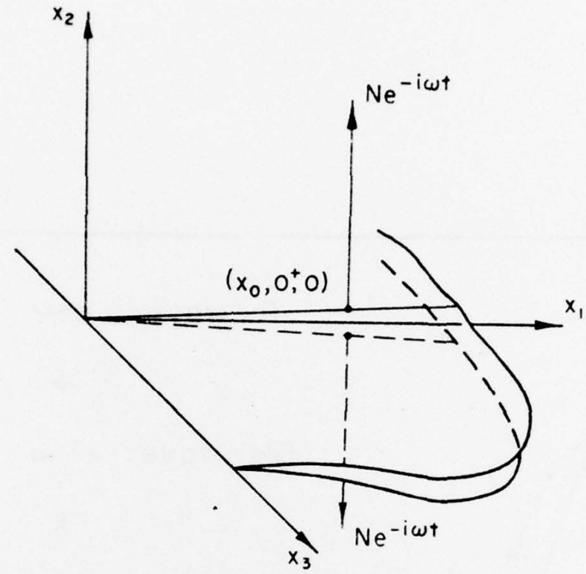

Fig. 6   Point loads applied on crack faces

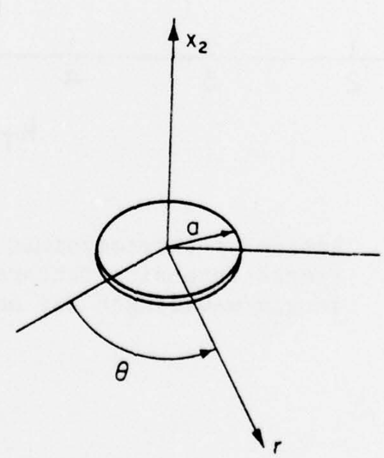

Fig. 7   Penny-shaped crack

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NU-SML TR No. 77-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Ray Theory for Elastodynamic Stress Intensity factors		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. D. Achenbach and A. K. Gautesen		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0063
9. PERFORMING ORGANIZATION NAME AND ADDRESS Northwestern University Department of Civil Engineering Evanston, Illinois 60201		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NA
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Structural Mechanics Program Department of the Navy, Arlington, VA 22217		12. REPORT DATE March, 1977
		13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

crack
stress intensity factors
elastodynamics
waves

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Elastodynamic stress intensity factors generated by the interaction of wave motions with a crack are analyzed. It is shown that in an asymptotic approximation for high frequencies, the stress intensity factors at the edge of a crack are related to the fields of incident rays by a matrix of stress intensity factor coefficients, which can be computed from canonical solutions. The canonical solutions are the fields describing diffraction by a semi-infinite crack of plane body waves and plane surface waves, which are incident under an arbitrary angle with the edge of the crack. Several applications of the theory are presented.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)